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Plenary Talks

On square-difference factor absorbing ideals of commutative rings

Ayman Badawi

Let R be a commutative ring with 1 not equal 0. A proper ideal I of R is a square-difference factor absorbing ideal (sdf-absorbing ideal) of R if whenever $a^2 - b^2$ in I for nonzero $a, b \in R$, then $a + b \in I$ or $a - b \in I$. In this paper, we introduce and investigate sdf-absorbing ideals.

Keywords: Commutative rings
General area of research: Algebra
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Determination of some graph dimensions obtained from special algebraic structures

Ahmet Sinan Çevik

Department of Mathematics, Faculty of Science, Selcuk University, Campus, 42130, Konya, Turkey; sinan.cevik@selcuk.edu.tr, www.ahmetsinancevik.com

The determination of certain graph dimensions derived from specific structures is the objective of this study. The graphs [8] consider in this study will be simple and connected.
In detail, the main purpose of this talk is to partially answer the open problem "characterizing all graphs having infinite multiset dimensions" via the graphs obtained from special minimal (while inefficient or not) monoid presentations.
For the algebraic part of this study, we may refer [1,2]. On the other hand, the whole requested details such as variants of definitions and their properties etc. about the special graph dimensions can be found, for instance, in [5-7,9-13]

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Keywords: Minimal presentation, Graphs, Graph Dimensions
General area of research: Algebra
IIIMT25-ID 1703

Generalized bi-skew Lie (Jordan)-type Derivations on Algebras

Mohammad Ashraf

Professor (Retd.),Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India; mashraf80@gmail.com

Let $\mathcal{A}$ be a $*$-algebra over a commutative unital ring $\mathcal{R}$. An $\mathcal{R}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is said to be a $*$-*derivation* if $\delta(xy) = \delta(x)y + x\delta(y)$ and $\delta(x^*) = \delta(x)^*$ hold for all $x, y \in \mathcal{A}$. Let $[x, y]_*^* = xy^* - yx^*$ denote the *bi-skew Lie product* of $x, y \in \mathcal{A}$. For any $x_1, x_2, \ldots, x_n \in \mathcal{A}$ and integer $n \geq 2$, define $p_1(x_1) = x_1$, $p_2(x_1, x_2) = [x_1, x_2]_*^*$ and $p_n(x_1, x_2, \ldots, x_n) = [p_{n-1}(x_1, x_2, \ldots, x_{n-1}), x_n]_*^*$. For integer $n \geq 2$, the polynomial

$$p_n(x_1, x_2, \ldots, x_n)$$

is called the *bi-skew Lie n-product* of elements $x_1, x_2, \ldots, x_n \in \mathcal{A}$. An $\mathcal{R}$-linear mapping $\mathcal{L} : \mathcal{A} \to \mathcal{A}$ is said to be a *bi-skew Lie n-derivation* if

$$\mathcal{L}(p_n(x_1, \ldots, x_n)) = \sum_{i=1}^{n} p_n(x_1, \ldots, x_{i-1}, \mathcal{L}(x_i), x_{i+1}, \ldots, x_n)$$

holds for all $x_1, x_2, \ldots, x_n \in \mathcal{A}$. Assume that $\mathcal{G}_{\mathcal{L}} : \mathcal{A} \to \mathcal{A}$ is an $\mathcal{R}$-linear mapping and $\mathcal{L}$ is a bi-skew Lie n-derivation on $\mathcal{A}$. Then $\mathcal{G}_{\mathcal{L}}$ is called a *generalized bi-skew Lie n-derivation* with associated bi-skew the Lie n-derivation $\mathcal{L}$ if $\mathcal{G}_{\mathcal{L}}(p_n(x_1, \ldots, x_n)) = p_n(\mathcal{G}_{\mathcal{L}}(x_1), x_2, \ldots, x_n) + \sum_{i=2}^{n} p_n(x_1, \ldots, x_{i-1}, \mathcal{L}(x_i), x_{i+1}, \ldots, x_n)$ holds for all $x_1, x_2, \ldots, x_n \in \mathcal{A}$. In the above definitions, if we replace the bi-skew Lie product $[x, y]_*^* = xy^* - yx^*$ by the *bi-skew Jordan product* $x \circ y = xy^* + yx^*$, then the mappings $\mathcal{L}$ and $\mathcal{G}_{\mathcal{L}}$ are known as a *bi-skew Jordan n-derivation* and a *generalized bi-skew Jordan n-derivation*, respectively.

Determining the Lie (Jordan) structure of a $*$-algebra is one of the most important topics in algebras and has been studied extensively by many authors (see [1-4] and references therein). Kong and Zhang, in [5] proved that every nonlinear bi-skew Lie 2-derivation on a factor von Neumann algebra $\mathcal{A}$ with $\dim(\mathcal{A}) \geq 2$ is an additive $*$-derivation. The structure of generalized bi-skew Jordan n-derivations and some related mappings have been studied in [3]. In the present talk, the afore-mentioned developments will be discussed in details together with some potential future research problems in this direction.

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Keywords: $*$-algebras; factor von Neumann algebras; $*$-derivations; bi-skew Lie n-derivations; bi-skew Jordan n-derivations; generalized bi-skew Lie n-derivations

General area of research: Algebra
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The primeness of noncommutative polynomials on prime rings

M. Tamer Koşan

Faculty Sciences, Department of Mathematics, Gazi University, Ankara, Turkey; mtamerkosan@gazi.edu.tr, tkosan@gmail.com

This talk is based on the paper [4] with same title which appeared in Journal Algebra and Applications , May 2024.
Throughout this presentation, rings R are always associative but are not necessarily with unity. We let $Z(R)$ denote the center of R. For $a, b \in R$, let $[a, b] := ab - ba$, the additive commutator of a and b. Given additive subgroups A, B of R, we denote AB (respectively, $[A, B]$) the additive subgroup of R generated by all elements ab (respectively, $[a, b]$) for $a \in A$ and $b \in B$.
Let R be an algebra over K, where K is a unital commutative ring. By a polynomial $f(X_1, \ldots, X_t)$, we always mean that $f(X_1, \ldots, X_t)$ is a polynomial over K in noncommutative variables $X_1, \ldots, X_t$ and it has zero constant term. A polynomial $f(X_1, \ldots, X_t))$ over K is called a polynomial identity (PI) for R if $f(x_1, \ldots, x_t) = 0$ for all $x_i \in R$. The polynomial $f(X_1, \ldots, X_t)$ is called central-valued on R if $f(x_1, \ldots, x_t) \in Z(R)$ for all $x_1, \ldots, xt \in R$. Given an additive subgroup A of R, let $f(A)$ denote the additive subgroup of R generated by all elements $f(x_1, \ldots, x_t)$ for all $x_1, \ldots, xt \in A$.
We first discuss the following observation which gives the primeness of non-central polynomials on algebras. Precisely,

Theorem A: Let R be an algebra over K, and let $f(X_1, \ldots, X_t)$ be a noncommutative polynomial over K, which is not central-valued on R. Then the following are equivalent:
(i) R is a prime ring;
(ii) Given $a, b \in R$, if $af(x_1, \ldots, x_t)b = 0$ for all $x_i \in R$ then either $a = 0$ or $b = 0$.
Roughly speaking, Theorem A means that a polynomial $f(X_1, \ldots, X_t)$ is "prime" if and only if its image (as a function on n copies of R) is "prime".
In a recent paper [3], Calareanu-Lee-Matczuk studied the notion of X-primeness of rings (see also [2] for unit-semiprime rings). Let R be a ring with a subset X. The ring R is called X-prime if, for $a, b \in R$, $aXb = 0$ implies that either $a = 0$ or $b = 0$. Clearly, every X-prime ring is itself prime. From the view of point, we can restate Theorem A as follows.

Theorem B: Let R be an algebra over K, and let $f(X_1, \ldots, X_t)$ be a polynomial over K, which is not central-valued on R. Then R is a prime ring iff it is $f(R)$-prime.
Theorem C: Let R be an algebra, ρ a right ideal of R, and $f(X_1, \ldots, X_t)$ be a polynomial. Then R is $f(\rho)$-prime iff it is $f(R)$-prime and $Ann^l_R(\rho) = 0$.

Here, we remark that Theorem C is the one-sided version of Theorem B.
Let R be a prime ring, and let U denote the maximal right ring of quotients of R. The center of U, denoted by C, is called the extended centroid of R. It is well known that U is also a prime ring and C is a field (see [1] for details). Also, R is called centrally closed if $R = RC$. In particular, RC is always a centrally closed prime algebra over C.
Basing on Theorem A, we prove its one-sided version.
Theorem D: Let R be a prime ring, ρ a right ideal of R, $f(X_1, \ldots, X_t)$ a noncommutative polynomial over C, which is not a PI for ρ, and $a, b \in R \setminus \{0\}$. Then $af(x_1, \ldots, x_t)b = 0$ for all $x_i \in \rho$ if and only if one of the following hold:
(i) $a\rho = 0$;
(ii) $\rho C = eRC$ for some idempotent $e \in RC$ and $b \in \rho C$ such that either $f(\rho)\rho = 0$ or $f(X_1, \ldots, X_t)$ is central-valued on $eRCe$ and $ab = 0$.
Higher commutators of a given ring due to Lanski [5] are defined as follows.
Definition:

1. R is a higher commutator of R with length 1.
2. If U and V higher commutators of R with lengths s, t, respectively, then $[U, V]$ is also a higher commutator of R with length $s+t$.
3. Every higher commutator of R is obtained from (1) and (2) inductively.

Given a higher commutator H of R, the weight of H, denoted by $w(R)$, is defined as the minimal length of its all possible expressions.
If $H = [R, [R, R]]$, we can choose $f = [X_1, [X_2, X_3]]$ such that $f(R) = H$. Clearly, it is true for arbitrary higher commutator of R.
In the rest of the talk, we will apply Theorem D to the case that the additive subgroup of R generated by the image of $f(X_1, \ldots, X_t)$ on a right ideal is a higher commutator. Precisely, we will focus on the following problem.
Problem: Let R be a prime ring, ρ a nonzero right ideal of R, H a higher commutator of ρ and $a, b \in R$. Characterize a, b, and H if $aHb = 0$.

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General area of research: Algebra
IIIMT25-ID 1705

On Certain Additive Maps and the Subrings They Generate in a Prime Ring

Vincenzo De Filippis

Department of Engineering, University of Messina, Italy; defilippis@unime.it

Let R be a ring and f an additive map of R. How does one measure the size of $f(R)$? One way is to look at how large $f(R)$, the subring generated by $f(R)$, turns out to be. A subring of a ring is considered 'large' if it contains an ideal (right, left, or two-sided) of the ring itself. In this presentation, we aim to analyze the subrings generated by appropriate sets constructed using some of the most commonly used additive maps, working in a context related to prime rings of characteristic different from 2. We will reference some of the main results from the literature, present a few new ones, and provide some ideas for future research, suggesting some open problems to address.

Keywords: Prime rings, Generalized derivations, Generalized Skew derivations, Generalized homoderivations
General area of research: Noncommutative Ring Theory
IIIMT25-ID 1706

The First General Zagreb Index of a Graph for the Ring of Integers Modulo $p^k qr$

Nor Haniza Sarmin[1], Ghazali Semil Ismail[1,2], Nur Idayu Alimon[2]

[1]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia; nhs@utm.my, ghazali85@graduate.utm.my
[2]Mathematical Sciences Studies, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA Johor Branch, Pasir Gudang Campus, 81750, Masai, Johor, Malaysia; idayualimon@uitm.edu.my

A topological index, also known as a connectedness index, is a numerical descriptor derived from the molecular graph of a chemical compound, providing an accurate representation of its topological structure. Topological indices are classified based on degree, distance, eigenvalue, matching, and mixed, and in this article, we focus on calculating and analysing degree-based topological index, namely the first general Zagreb index. The first general Zagreb index of a graph is defined as the sum of the degrees of all vertices in the graph, each raised to the power of δ, where δ is any nonzero real number. Suppose we have a simple graph with the set of vertices and edges. The zero divisor graph of a commutative ring is a graph whose vertices correspond to the zero divisors of the ring. In this graph, two distinct vertices are adjacent if and only if their product is zero. In this research, the general formulas of the first general Zagreb index of the zero divisor graph for the ring of integers modulo $p^k qr$, where p, q and r are distinct primes and $k \in \mathbb{N}$, for cases $\delta = 1$, 2 and 3 are determined. Some examples are provided to demonstrate the results.

Introduction and Preliminaries

Topological indices, numerical parameters derived from the graph representation of a molecule, have become indispensable tools in cheminformatics and quantitative structure-property relationship studies, offering a succinct yet powerful means of characterising molecular structures and predicting their physicochemical properties [1]. The concept of a zero divisor graph provides a valuable bridge between ring theory and graph theory by allowing algebraic properties to be studied through graphical structures. In recent years, a substantial body of research has focused on graph-theoretical topological indices, including studies specifically addressing zero divisor graphs of commutative rings. In this section, we provide definitions of the zero divisor graph of a commutative ring and the first general Zagreb index.

Definition 1. [2] *The zero divisor graph of a commutative ring R, denoted by $\Gamma(R)$, is the undirected graph with vertex set $Z(R)$ and two distinct vertices u and v are adjacent if $uv = 0$.*

Following the introduction of zero divisor graphs, extensive research has been conducted on their structural and combinatorial properties within the context of commutative rings, including studies on upper dimension and bases [3], metric dimension [4], eigenvalues [5], and other graph parameters [6].

Definition 2. [7] *The first general Zagreb index,*

$$R_\delta^0 = \sum_{u \in V(\Gamma)} deg\,(u)^\delta\,,$$

where δ is an arbitrary real number.

For recent studies on the computation of topological indices for the total graph and the zero divisor graph of a commutative ring, readers are referred to [8–11]. Notably, in 2023, Mondal et al. [12] computed several degree-distance-based and distance-based topological indices for the zero divisor graphs of the ring $\mathbb{Z}_{p^k}$. Throughout this paper, we derive the general formulas of the first general Zagreb index of the zero divisor graph for the ring of integers modulo $p^k qr$, where p, q and r are distinct primes and $k \in \mathbb{N}$. The computations are carried out for the cases $\delta = 1$, 2 and 3.

Main Results and Discussion

The following propositions are presented for the set of all zero divisors and the number of zero divisors in the commutative ring $\mathbb{Z}_{p^k qr}$.

Proposition 3. *The set of all zero divisors in the ring $\mathbb{Z}_{p^k qr}$ is given by $Z\left(\mathbb{Z}_{p^k qr}\right) = \{p, 2p, 3p, \ldots, p\left(p^{k-1}qr - 1\right)\} \cup \{q, 2q, 3q, \ldots, q\left(p^k r - 1\right)\} \cup \{r, 2r, 3r, 4r, \ldots, r\left(p^k q - 1\right)\}$.*

Proposition 4. *The number of zero divisors in the commutative ring $\mathbb{Z}_{p^k qr}$ is given by $\left|Z\left(\mathbb{Z}_{p^k qr}\right)\right| = p^{k-1}(qr - r - q + 1) + p^k(r + q - 1) - 1$.*

Initially, the degree of a vertex in the zero divisor graph for the ring $\mathbb{Z}_{p^k qr}$ is analysed through seven distinct cases, as outlined in Propositions 2.3 to 2.9, presented in the following sections.

Proposition 5. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ with $gcd\big(a, p^k qr\big) = p^i$ for $i = 1, 2, 3, \ldots, k$. Then, $deg(a) = p^i - 1$.*

Proposition 6. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ with $gcd\big(a, p^k qr\big) = q$. Then, $deg(a) = q - 1$.*

Proposition 7. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ with $gcd\big(a, p^k qr\big) = r$. Then, $deg(a) = r - 1$.*

Proposition 8. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ with $gcd\big(a, p^k qr\big) = p^i q$ for $i = 1, 2, 3, \ldots, k$. Then, $deg(a) = p^i q - 1$.*

Proposition 9. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ with $gcd\big(a, p^k qr\big) = p^i r$ for $i = 1, 2, 3, \ldots, k$. Then, $deg(a) = p^i r - 1$.*

Proposition 10. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ with $gcd\big(a, p^k qr\big) = qr$. Then, $deg(a) = qr - 1$.*

Proposition 11. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ with $gcd\big(a, p^k q^r\big) = p^i qr$. Then,*

$$deg(a) = \begin{cases} p^i qr - 1, & \text{for } i \leq \left\lfloor \frac{k-1}{2} \right\rfloor, \\ p^i qr - 2, & \text{for } i > \left\lfloor \frac{k-1}{2} \right\rfloor. \end{cases}$$

In the second procedure, the number of vertices in the zero divisor graph for the ring $\mathbb{Z}_{p^k qr}$ corresponding to each degree is categorised into seven cases, as detailed in Propositions 2.10 tO 2.16.

Proposition 12. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ where $gcd\big(a, p^k qr\big) = p^i$. Then,*

$$\left|V\left(\Gamma\left(\mathbb{Z}_{p^k qr}\right)\right)\right| = \begin{cases} \left(p^{k-i} - p^{k-(i+1)}\right)(q-1)(r-1), & \text{for } 1 \leq i \leq k-1, \\ p^{k-i}(q-1)(r-1), & \text{for } i = k. \end{cases}$$

Proposition 13. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ where $gcd\big(a, p^k qr\big) = q$. Then,*

$$\left|V\left(\Gamma\left(\mathbb{Z}_{p^k qr}\right)\right)\right| = \left(p^k - p^{k-1}\right)(r-1).$$

Proposition 14. *Let $a \in Z\left(\mathbb{Z}_{p^k qr}\right)$ where $gcd\big(a, p^k qr\big) = r$. Then,*

$$\left|V\left(\Gamma\left(\mathbb{Z}_{p^k qr}\right)\right)\right| = \left(p^k - p^{k-1}\right)(q-1).$$

Proposition 15. *Let* $a \in Z\left(\mathbb{Z}_{p^kqr}\right)$ *where* $gcd\big(a, p^kqr\big) = p^iq$. *Then,*

$$\left|V\left(\Gamma\left(\mathbb{Z}_{p^kqr}\right)\right)\right| = \begin{cases} \left(p^{k-i} - p^{k-(i+1)}\right)(r-1), & \text{for } 1 \leq i \leq k-1, \\ p^{k-i}(r-1), & \text{for } i = k. \end{cases}$$

Proposition 16. *Let* $a \in Z\left(\mathbb{Z}_{p^kqr}\right)$ *where* $gcd\big(a, p^kqr\big) = p^ir$. *Then,*

$$\left|V\left(\Gamma\left(\mathbb{Z}_{p^kqr}\right)\right)\right| = \begin{cases} \left(p^{k-i} - p^{k-(i+1)}\right)(q-1), & \text{for } 1 \leq i \leq k-1, \\ p^{k-i}(q-1), & \text{for } i = k. \end{cases}$$

Proposition 17. *Let* $a \in Z\left(\mathbb{Z}_{p^kqr}\right)$ *where* $gcd\big(a, p^kqr\big) = qr$. *Then,*

$$\left|V\left(\Gamma\left(\mathbb{Z}_{p^kqr}\right)\right)\right| = \left(p^k - p^{k-1}\right).$$

Proposition 18. *Let* $a \in Z\left(\mathbb{Z}_{p^kqr}\right)$ *where* $gcd\big(a, p^kqr\big) = p^iqr$. *Then*

$$\left|V\left(\Gamma\left(\mathbb{Z}_{p^kqr}\right)\right)\right| = \left(p^{k-i} - p^{k-(i+1)}\right)$$

for $1 \leq i \leq k-1$.

Lastly, the general formulas of the first general Zagreb index of the zero divisor graph for the ring $\mathbb{Z}_{p^kqr}$, denoted as $R^0_\delta\left(\Gamma\left(\mathbb{Z}_{p^kqr}\right)\right)$ when $\delta = 1, 2$ and 3 are determined. To obtain the general formulas, Propositions 2.1 to 2.16 are systematically incorporated into the definition of the topological index.

Theorem 19. *The first general Zagreb index of the zero divisor graph for the ring* $\mathbb{Z}_{p^kqr}$ *when* $\delta = 1$,

$$\begin{aligned} R^0_1\left(\Gamma\left(\mathbb{Z}_{p^kqr}\right)\right) &= (q-1)(r-1)\left(k\left(p^k - p^{k-1}\right) + 2p^{k-1}(p-1)\right) \\ &+ (r-1)\left(q\left(p^k - p^{k-1}\right)(k-1) - p^{k-1} + p^kq\right) \\ &+ (q-1)\left(r\left(p^k - p^{k-1}\right)(k-1) - p^{k-1} + p^kr\right) \\ &+ \left(p^k - p^{k-1}\right)\left(qrk - 1 - \left(\frac{p^{k-1} - p^{\left\lfloor \frac{k-1}{2} \right\rfloor}}{p^{\left\lfloor \frac{3(k-1)}{2} \right\rfloor}(p-1)}\right)\right) - p^{k-1} + 1. \end{aligned}$$

Theorem 20. *The first general Zagreb index of the zero divisor graph for the ring* $\mathbb{Z}_{p^kqr}$ *when* $\delta = 2$,

$$\begin{aligned} R^0_2\left(\Gamma\left(\mathbb{Z}_{p^kqr}\right)\right) &= (q-1)(r-1)\Big[p^{k-1}(p^k - p) - 2(p^k - p^{k-1})(k-1) + p^{k-1} - 1 \\ &+ (p^k-1)^2 + p^{k-1}(q+r-2)\Big] + (r-1)\Big[p^{k-1}q^2(p^k - p) \\ &- 2q(p^k - p^{k-1})(k-1) + p^{k-1} - 1 + (p^kq-1)^2\Big] \\ &+ (q-1)\Big[p^{k-1}r^2(p^k - p) - 2r(p^k - p^{k-1})(k-1) + p^{k-1} - 1 \\ &+ (p^kr-1)^2\Big] + (p^k - p^{k-1})\Big[(qr-1)^2 + q^2r^2\left(\frac{p^k - p}{p-1}\right) \\ &- 2qr(k-1) + \frac{p^{k-1} - 1}{p^k - p^{k-1}} + \frac{3\left(p^{k-1} - p^{\left\lfloor \frac{k-1}{2} \right\rfloor}\right)}{p^{\left\lfloor \frac{3(k-1)}{2} \right\rfloor}(p-1)} - 2qr\left\lceil \frac{k-1}{2} \right\rceil\Big]. \end{aligned}$$

Theorem 21. *The first general Zagreb index of the zero divisor graph for the ring $\mathbb{Z}_{p^kqr}$ when $\delta = 3$,*

$$\begin{aligned}
R_3^0(\Gamma(\mathbb{Z}_{p^kqr})) = (q-1)(r-1) & \left[\frac{p^{k+1}(p^{2(k-1)}-1)}{p+1} - 3p^{k-1}(p^k - p) \right. \\
& + 3(p^k - p^{k-1})(k-1) - p^{k-1} + 1 + (p^k-1)^3 + p^{k-1}(p-1)(q-1)^2 \\
& \left. + p^{k-1}(p-1)(r-1)^2 \right] \\
& + (r-1) \left[\frac{p^{k+1}q^3(p^{2(k-1)}-1)}{p+1} - 3p^{k-1}q^2(p^k - p) \right. \\
& \left. + 3q(p^k - p^{k-1})(k-1) - p^{k-1} + 1 + (p^kq-1)^3 \right] \\
& + (q-1) \left[\frac{p^{k+1}r^3(p^{2(k-1)}-1)}{p+1} - 3p^{k-1}r^2(p^k - p) \right. \\
& \left. + 3r(p^k - p^{k-1})(k-1) - p^{k-1} + 1 + (p^kr-1)^3 \right] \\
& + (p^k - p^{k-1}) \left[(qr-1)^3 + p^2q^3r^3 \left(\frac{p^{2(k-1)}-1}{p^2-1} \right) - 3q^2r^2 \left(\frac{p^k - p}{p-1} \right) \right. \\
& + 3qr(k-1) - \frac{p^{k-1}-1}{p^k - p^{k-1}} - \frac{3p^{\lfloor \frac{k+1}{2} \rfloor} q^2r^2 \left(p^{\lceil \frac{k-1}{2} \rceil} - 1 \right)}{p-1} \\
& \left. - \frac{7\left(p^{k-1} - p^{\lfloor \frac{k-1}{2} \rfloor}\right)}{p^{\lfloor \frac{3(k-1)}{2} \rfloor}(p-1)} + 9qr \left\lceil \frac{k-1}{2} \right\rceil \right].
\end{aligned}$$

Conclusion

In this paper, several properties such as the set of vertices representing zero divisors, the degree of each vertex and the number of vertices of the zero divisor graph for the ring of integers modulo p^kqr are determined. Based on these structural characteristics, general formulas for the first general Zagreb index of the graph are successfully established.

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Quiver Representations in Neural Network and

Intan Muchtadi-Alamsyah

Algebra Research Group, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung; ntan@itb.ac.id

Neural networks, consisting of interconnected layers of neurons, have found widespread applications in various domains, such as image recognition, stock market prediction, and speech recognition. However, understanding the structure and characteristics of neural networks can be challenging. In this talk, we will explore how quiver representations over a valuable framework for describing neural networks.

Topological data analysis (TDA) is a rapidly evolving field that employs topological methods to analyze complex datasets. Its objective is to reveal the underlying structure of data by identifying topological features such as connected components and voids. Quiver representations have emerged as a promising approach within TDA, leveraging directed graphs to encode information about a system. Their effectiveness in representing topological structures, including persistence diagrams, has increased popularity.

This talk will also provide an overview of quiver representations in the context of topological data analysis. By understanding the role and significance of quiver representations, we can enhance our understanding of neural networks and apply this knowledge to various domains where these networks are utilized.

Keywords: Topological Data Analysis
General area of research: Mathematics
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The Zariski Closure Conjecture for exponentially Lie groups

Ali Baklouti

Faculté des Sciences de Sfax, Département de Mathématiques,Sfax Tunisie; ali.baklouti@usf.tn

We will begin by defining the Zariski Closure Conjecture for coadjoint orbits of exponentially solvable Lie groups, examining some solved cases, and addressing the ongoing challenges in fully resolving the conjecture. I will then introduce new approaches to exploring the relationship between generating families of primitive ideals and the set of polynomials that vanish on the associated coadjoint orbits, ultimately aiming to advance toward a solution to the conjecture.

Keywords: Zariski Closure Conjectur
General area of research: Mathematics
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Invited Talks

Chain and Distributive Coalgebras

Christian Lomp

Department of Mathematics, Faculty of Sciences, University of Porto, Porto, Portugal; clomp@fc.up.pt

In this talk we will see that coalgebras whose lattice of right coideals is distributive are coproducts of coalgebras whose lattice of right coideals is a chain. Those chain coalgebras are characterized as finite duals of Noetherian chain rings whose residue field is a finite dimensional division algebra over the base field. Infinite dimensional chain coalgebras are finite duals of left Noetherian chain domains. Given any finite dimensional division algebra D and D-bimodule structure on D, we construct a chain coalgebra as a cotensor coalgebra. Moreover if D is separable over the base field, every chain coalgebra of type D can be embedded in such a cotensor coalgebra. As a consequence, cotensor coalgebras arising in this way are the only infinite dimensional chain coalgebras over perfect fields. Finite duals of power series rings with coeficients in a finite dimensional division algebra D are further examples of chain coalgebras, which also can be seen as tensor products of D^*, and the divided power coalgebra and can be realized as the generalized path coalgebra of a loop. If D is a central division algebra, any chain coalgebra is a subcoalgebra of the finite dual of D[[x]]. (This talk is based on a joined work with Alveri Sant'ana)

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Keywords: Coalgebras, chain rings, distributivity
General area of research: Algebra
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On certain Identities with automorphisms in prime and semiprime rings

Nadeem ur Rehman

Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India; nu.rehman.mm@amu.ac.in

Let R be a prime ring with center Z and maximal right ring of quotients $Q = Q_{mr}(R)$. Note that Q is also a prime ring and the center C of Q, which is called the extended centroid of R, is a field. Moreover, $Z \subseteq C$. It is well known that any automorphism of R can be uniquely extended to an automorphism of Q. An automorphism α of R is called Q-inner if there exists an invertible element $g \in Q$ such that $\alpha(x) = gxg^{-1}$ for all $x \in R$. Otherwise, α is called Q-outer. We denote by G the group of all automorphisms of R and by A_i the group consisting of all Q-inner automorphisms of R. Recall that a subset $\mathfrak{A}$ of G is said to be independent (modulo A_i) if for any $a_1, a_2 \in \mathfrak{A}$, $a_1 a_2^{-1} \in A_i$ implies $a_1 = a_2$. For instance, if a is an outer automorphism of R, then 1 and a are independent (modulo A_i). In the year 2000, Carini and De Filippis [1] studied the power-centralizing derivations on noncentral Lie ideals of prime rings. They proved that, if char$(R) \neq 2$ and $[d(x), x]^n \in Z$ for all x in a non-central Lie ideal L of R, then R satisfies s_4, the standard identity in four variables. Recently, Wang [2] obtained similar result for automorphisms of prime rings. To be more specific, Wang proved the following: Let R be a prime ring with center Z, L be a non-central Lie ideal of R and α be a nontrivial automorphism of R such that $[\alpha(u), u]^n \in Z$ for all $u \in L$. If either char$(R) > n$ or char$(R) = 0$, then R satisfies s_4.

On the other hand, the property $x^n = x$ has been among the favorites of many ring theorists over the last many decades since Jacobson [3] first studied the commutativity of rings satisfying this condition in order to generalize the classical Wedderburn theorem. Further, Bell and Ligh [4] obtained a direct sum decomposition of a ring satisfying the property $xy = (xy)^2 f(x, y)$, where $f(x, y) \in \mathbb{Z} < x, x >$, the ring of polynomials in two non-commuting indeterminates. Later, Ashraf [5] established a decomposition theorem for rings satisfying $yx = x^m f(xy) x^n$ or $xy = x^m f(xy) x^n$, where m, n are non-negative integers and $f(x) \in x^2\mathbb{Z}[x]$, which allows us to determine the commutativity of R. Now in this perspective and inspired by Wang works, in the present talk we discussed the action of automorphisms on Lie ideals of prime ring and semiprime rings.

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Keywords: Prime and semiprime rings
General area of research: Algebra
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A study on the hulls of constacyclic codes over $R_{m,q}$

Om Prakash, Indibar Debnath

Department of Mathematics, Indian Institute of Technology Patna, Bihta, Patna, Bihar, India; om@iitp.ac.in

The hull of a linear code is the intersection of the code with its dual. Assmus Jr. and Key [1] introduced the concept of the hull in 1990. It has a crucial role in determining the complexity of the algorithms used to check the permutation equivalence of two linear codes or determine the automorphism group of a linear code. Further, good entanglement-assisted quantum error-correcting codes are obtained from the hulls of linear codes. Interestingly, the linear codes with trivial hulls are called the linear complementary dual (LCD) codes that are used to protect crypto-systems. All these applications of hulls have intrigued researchers to study the hulls and their properties extensively. In this talk, we consider the ring $R_{m,q} = \frac{\mathbb{F}_q[u]}{\langle u^m - u \rangle}$ and define the Galois inner product over this ring. Then, we study the Galois duals of constacyclic codes over the ring and propose a formula for the Galois hull dimensions of constacyclic codes. Furthermore, we present some results on constacyclic codes over $R_{m,q}$ to be Galois LCD and give a few examples of Galois LCD codes.

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Keywords: Constacyclic code, Galois hull, Galois LCD code
General area of research: Coding theory
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Jordan Derivations on Modules over Associative Rings: Equivalence, Structure, and Applications

Indah Emilia Wijayanti

Department of Mathematics Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Yogyakarta, Indonesia; ind_wijayanti@ugm.ac.id

This paper investigates the structure and properties of Jordan derivations on modules over associative rings. A Jordan derivation is a linear map that satisfies the Leibniz rule for the Jordan product $a \circ b = ab + ba$, generalizing the notion of classical derivations. We explore the relationship between Jordan derivations and ordinary derivations in the context of modules, focusing on conditions under which these two classes of maps coincide. By leveraging the algebraic structure of modules and their underlying rings, we establish sufficient criteria for a Jordan derivation to be a derivation, particularly in the setting of prime and semiprime modules. Key results include the demonstration that, under certain faithfulness and torsion-free conditions, every Jordan derivation on a module reduces to a derivation.

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Keywords: Jordan derivation, Leibniz rule, (semi)prime module, torsion-free module
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Quasi-Duo Modules

A. Çiğdem Özcan

Department of Mathematics, Hacettepe University, 06800 Beytepe, Ankara, Türkiye; ozcan@hacettepe.edu.tr

A ring R is said to be a left quasi-duo ring if its maximal left ideals are two-sided, or equivalently every maximal left ideal is fully invariant in R. The concept of quasi-duo modules arises as a natural generalization of quasi-duo rings and has been of interest to many authors in the literature. In this article, quasi-duo modules are investigated in detail and their relations with some other classes of modules are examined.
Joint work with Mauricio Gabriel Medina Bárcenas

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General area of research: Algebra
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Additives mappings on prime and semiprime rings: a survey

Abdellah Mamouni

Moulay Ismail University, Meknes Morocco; a.mamouni.fste@gmail.com

The purpose of this work is to prove some results concerning some Jordan derivation and left Jordan derivation on prime and semi-prime rings. Moreover, we provide examples to show that the assumed restrictions cannot be relaxed.

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General area of research: Algebra
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Coding Theory: Past, Present, and Future

Nuh Aydın

Department of Mathematics and Statistics, Kenyon College, Gambier, OH, USA; aydinn@kenyon.edu

Coding theory is among the most elegant and useful applications of algebra. This relatively young discipline benefits much from algebraic tools and generates interesting problems in pure algebra. In this talk, we give a brief introduction to coding theory including its history, current and future research questions.

Keywords: Coding theory
General area of research: Algebra
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p-adic integrals involving special numbers on p-adic integers with their ideals and additive cosets

Yilmaz Simsek

Department of Mathematics, Faculty of Science University of Akdeniz, Antalya-Turkey, ysimsek@akdeniz.edu.tr

The aim of this survey is to analyze the results given in both "[8] and [9]", which include new p-adic integral formulas in recent years, and to give new formulas and relations related to this subject. In addition to these, it is to give their applications and to demonstrate their current usage, not only in the scope of measure theorem, distribution theory involving the Haar distribution, and p-adic integrals, but also by blending them with algebraic structures and certain family of p-adic zeta functions.

Introduction

Let $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, the set of integers, the set of rational numbers, the set of real numbers and the set of complex numbers, respectively. Additionally, let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Here we assume that p be an odd prime number. For $m \in \mathbb{N}$, definition of $ord_p(m)$: $ord_p(m)$ is the greatest integer k ($k \in \mathbb{N}_0$) such that p^k divides m in $\mathbb{Z}$. For $m = 0$, assuming that $ord_p(m) = \infty$. For $x \in \mathbb{Q}$ with $x = \frac{a}{b}$ $(a, b \in \mathbb{Z})$, than $ord_p(x) = ord_p(\frac{a}{b}) = ord_p(a) - ord_p(b)$. Let $|.|_p$ is a map on $\mathbb{Q}$. $|.|_p$ is a norm over $\mathbb{Q}$. $|.|_p$ is defined by

$$|x|_p = \begin{cases} p^{-ord_p(x)} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

In order to explain any $x \in \mathbb{Q}$ with form $x = p^y \frac{x_1}{x_2}$ where $y, x_1, x_2 \in \mathbb{Z}$ and x_1 and x_2 are not divisible by p, one has $ord_p(x) = y$ and $|x|_p = p^{-y}$ (*cf.* [1-12]).

The set $\mathbb{Q}_p$ equipped with this norm $|x|_p$ is a topological completion of of set $\mathbb{Q}$. $\mathbb{C}_p$ is the field of p-adic completion of algebraic closure of $\mathbb{Q}_p$. $\mathbb{Z}_p$ is topological closure of $\mathbb{Z}$. $\mathbb{Z}_p$ is a set of p-adic integers. With the aid of the norm $|x|_p$, $\mathbb{Z}_p$ is defined by not only as follows: $\mathbb{Z}_p = \left\{x \in \mathbb{Q}_p : |x|_p \leq 1\right\}$, but also by the formal power series: If $x \in \mathbb{Z}_p$, then $x = \sum\limits_{j=0}^{\infty} a_j p^j$ where $0 \leq a_j < p$ with $j \in \mathbb{N}_0$ (*cf.* [1-12]).

Here we assume that $f : \mathbb{Z}_p \to \mathbb{C}_p$ is a uniformly differential function at a point $a \in \mathbb{Z}_p$.

The form of additive cosets of $\mathbb{Z}_p$ are given by means of the following sets: $p\mathbb{Z}_p = \left\{x \in \mathbb{Z}_p : |x|_p < 1\right\}$, which is a maximal ideal of $\mathbb{Z}_p$. For $j \in \{0, 1, \ldots, p^n - 1\}$, all additive cosets of $\mathbb{Z}_p$ are given by $p\mathbb{Z}_p, 1+p\mathbb{Z}_p, \cdots, p-1+p\mathbb{Z}_p$, where the set $j+p\mathbb{Z}_p$ is given by $j+p^n\mathbb{Z}_p = \left\{x \in \mathbb{Z}_p : |x-j|_p < p^{1-n}\right\}$ and $\mathbb{Z}_p = \cup_{j=0}^{p-1} (j + p\mathbb{Z}_p)$ (*cf.* [1-12]).

Every map μ from the set of intervals contained in X to $\mathbb{Q}_p$ for which

$$\mu\left(x + p^n\mathbb{Z}_p\right) = \sum_{j=0}^{p-1} \mu\left(x + jp^n + p^{n+1}\mathbb{Z}_p\right)$$

whenever $x + p^n\mathbb{Z}_p \subset X$, exists uniquely to a p-adic distribution on X. There are many examples for istributions. The firs important example is the Haar distribution, defined by $\mu_{Haar}\left(x + p^N\mathbb{Z}_p\right) := \mu_1(x) = \mu_1\left(x + p^N\mathbb{Z}_p\right) = \frac{1}{p^N}$ (*cf.* [1-12]).

Observe that, for a compact-open subset $\mathbb{X}$ of $\mathbb{Q}_p$, a p-adic distribution μ on $\mathbb{X}$ is a $\mathbb{Q}_p$-linear vector space homomorphism from the $\mathbb{Q}_p$-vector space of locally constant functions on $\mathbb{X}$ to $\mathbb{Q}_p$ (*cf.* [7]).

Let $\mathbb{K}$ be a field with a complete valuation and $C^1(\mathbb{Z}_p \to \mathbb{K})$ be a set of functions which have continuous derivative (see, for detail, [7]).
The Volkenborn integral (or p-adic bosonic integral) on $\mathbb{Z}_p$ is defined by

$$\int_{\mathbb{Z}_p} f(x)\, d\mu_1(x) = \lim_{N\to\infty} \frac{1}{p^N} \sum_{x=0}^{p^N-1} f(x), \tag{1}$$

where

$$\mu_1(x) = \frac{1}{p^N}$$

(*cf.* [1-12]).
Some properties of the Volkenborn integral (bosonic p-adic integral) are given as follows:
By applying the Volkenborn integral to the function $f(x) = \sum_{n=0}^{\infty} a_n \binom{x}{n} \in C^1(\mathbb{Z}_p \to \mathbb{K})$, one has the following well-known formula:

$$\int_{\mathbb{Z}_p} f(x)\, d\mu_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} a_n,$$

(*cf.* [7, p. 168-Proposition 55.3]).
Schikhof [7] gave the following integral formula for the Volkenborn integral:

$$\int_{\mathbb{Z}_p} f(x+1) d\mu_1(x) = \int_{\mathbb{Z}_p} f(x) d\mu_1(x) + f'(0), \tag{2}$$

where $f'(0) = f'(x)\,|_{x=0} = \frac{d}{dx}\{f(x)\}\,|_{x=0}$. By applying the Volkenborn integral to the following analytic function: $f : \mathbb{Z}_p \to \mathbb{K}$ with $f(x) = \sum_{n=0}^{\infty} a_n x^n$, we have

$$\int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} a_n x^n d\mu_1(x) = \sum_{n=0}^{\infty} a_n \int_{\mathbb{Z}_p} x^n d\mu_1(x) = \sum_{n=0}^{\infty} a_n B_n,$$

where B_n denotes the Bernoulli polynomials (*cf.* [1-12]).
We now study on the well-known properties of the multiplicative group of the primitive p^Nth roots of unity in $\mathbb{C}_p^* = \mathbb{C}_p \backslash \{0\}$.
Let C_{p^N} denote the multiplicative group of the primitive p^Nth roots of unity in $\mathbb{C}_p^* = \mathbb{C}_p \backslash \{0\}$.
The set $\mathbb{T}_p$ is defined by

$$\mathbb{T}_p = \left\{ \xi \in \mathbb{C}_p : \xi^{p^N} = 1,\, N \in \mathbb{N}_0 \right\} = \underset{N\geq 0}{\cup} \mathbb{C}_{p^N}$$

In [2], [?], [9], [12] and also the references cited in each of these earlier works, the p-adic Pontrjagin duality, the dual of $\mathbb{Z}_p$ is $\mathbb{T}_p = C_{p^\infty}$, the direct limit of cyclic groups C_{p^N} of order p^N with $N \geq 0$, with the discrete topology. $\mathbb{T}_p$ accept a natural $\mathbb{Z}_p$-module structure which can be written briefly as ξ^x for $\xi \in \mathbb{T}_p$ and $x \in \mathbb{Z}_p$. $\mathbb{T}_p$ are embedded discretely in $\mathbb{C}_p$ as the multiplicative p-torsion subgroup. If $\xi \in \mathbb{T}_p$, then

$$\vartheta_\xi : (\mathbb{Z}_p, +) \to (\mathbb{C}_p, .)$$

is the locally constant character, $x \to \xi^x$, which is a locally analytic character if $\xi \in \{\xi \in \mathbb{C}_p : ord_p(\xi - 1) >$
Consequently, it is well-known that ϑ_ξ has a continuation to a continuous group homomorphism from $(\mathbb{Z}_p, +)$ to $(\mathbb{C}_p, .)$ see also [1].

p-adic integral over subsets of $\mathbb{Z}_p$ and $\mathbb{C}_p$: Let $f \in C^1(\mathbb{Z}_p \to \mathbb{K})$. p-adic integral over $j + p^n\mathbb{Z}_p$, the cosets of ${}^n\mathbb{Z}_p$:

$$\int\limits_{j+p^n\mathbb{Z}_p} f(x)d\mu_1(x) = \int\limits_{p^n\mathbb{Z}_p} f(j+x)d\mu_1(x) = \frac{1}{p^n}\int\limits_{\mathbb{Z}_p} f(j+p^n x)d\mu_1(x) \tag{3}$$

(*cf.* [7, p. 175]). For instance,

$$\int\limits_{j+p^n\mathbb{Z}_p} x^m d\mu_1(x) = p^{n(m-1)} B_m\left(\frac{j}{p^n}\right) \tag{4}$$

(*cf.* [7, p. 175]). Let $\mathbf{R}_p = \mathbb{Z}_p \backslash p\mathbb{Z}_p$ and $f : \mathbf{R}_p \to \mathbb{Q}_p$ and a C^1-function and also $f(-x) = -f(x)$ with $x \in \boldsymbol{R}_p$. Thus $\int\limits_{\boldsymbol{R}_p} f(x)d\mu_1(x) = 0$ (*cf.* [7, p. 175]). The p-adic zeta function $\zeta_{p,j}(s)$ is defined on set $\boldsymbol{R}_p$ by

$$\int\limits_{\boldsymbol{R}_p} x^j \left(x^{p-1}\right)^s d\mu_1(x) = (j+(p-1)s)\,\zeta_{p,j}(s), \tag{5}$$

where $|s|_p < p^{\frac{p-2}{p-1}}$, $s \neq -\frac{j}{p-1}$ and $j \in \{0, 1, \ldots, p-2\}$, $p \neq 2$ (*cf.* [p. 187][7], [?]).
Substituting $s = n$ ($n \in \mathbb{N}$) into (5), one has

$$\zeta_{p,j}(n) = \frac{1}{j+(p-1)n}\int\limits_{\boldsymbol{R}_p} x^{j+n(p-1)} d\mu_1(x) = \frac{1}{j+(p-1)n} B_{j+n(p-1)} \tag{6}$$

where $n \in \mathbb{N}$ and $j \in \mathbb{N}_0$ (*cf.* [p. 187][7], [?]).
By using same mathods those of [8] and [9], we give some new p-adic integral formulas involving generating functions for certain classes of special numbers and polynomials, the Bernoulli numbers and polynomials, the Euler numbers and polynomials, the Stirling numbers, the Combinatorial numbers and sum. By combining the following integral, on the (maximal) ideals and also additive cosets of $\mathbb{Z}_p$,

$$\int\limits_{j+p^n\mathbb{Z}_p} f(x)d\mu_1(x)$$

with distribution theory related to the Haar distribution and p-adic integrals, study on p-adic zeta functions covering Bernoulli numbers.
Our future projects will be investigate applications results, which were given in [8], [9], and also other references.

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Keywords: p-adic Volkenborn integral, p-adic ideals, Bernoulli numbers
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The Generalized Hopfian Abelian Group in some Categories of Abelian Groups

Seddik Abdelalim [1]

[1]Department Mathematics and computer Science, University of Hassan II Casablanca, Casablanca ,Morocco; seddikabd@Hotmail.com

A group G is called a generalized Hopfian group if, for every surjective endomorphism f, Ker(f) is superfluous subgroup An abelian group A . In this paper We will characterize abelian group in category of Algebraically Compact abelian group and in category of divisible abelian group. We know that the p-component of generalized hopfian torsion abelian group is also generalized hopfian, but this result isn't true for any abelian group, for that we construct an generalized Hopfian abelian group but its the p-component of A isn't generalized Hopfian Hopfian abelian group

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The construction of few-weight minimal linear codes over finite fields

Mustafa Ali Çatalkaya[1], Ahmet Sınak [2]

[1] Mathematics, Necmettin Erbakan University, Konya, Türkiye; macatalkaya64@gmail.com
[2] Department of Management Information Systems, Akdeniz University, Antalya, Türkiye; ahmetsinak@akdeniz.edu.tr

Linear codes in coding theory are of great importance in various fields such as cryptographic systems, storage systems, and digital communication. In particular, few-weight minimal linear codes provide secure communication and storage for systems requiring privacy such as secret sharing schemes. In this work, we study the construction of few-weight minimal linear codes over the odd characteristic finite fields. Firstly, we construct a new family of three-weight linear codes by employing the defining set D_{01}. Secondly, we introduce a new construction method and obtain a new family of four-weight linear codes based on the defining set D_0. We calculate the Hamming weights and weight distributions of the obtained codes. Finally, we observe that these obtained codes are minimal.

For a prime number p and a positive integer m, the finite field with elements p^m is denoted by $\mathbb{F}_{p^m}$. The extension field $\mathbb{F}_{p^m}$ can be viewed as an m-dimensional vector space over $\mathbb{F}_p$, denoted by $\mathbb{F}_p^m$. The trace of $\alpha \in \mathbb{F}_{p^m}$ over $\mathbb{F}_p$ is defined as $\mathrm{Tr}_p^{p^m}(\alpha) = \alpha + \alpha^p + \alpha^{p^2} + \cdots + \alpha^{p^{m-1}}$, which is denoted by $\mathrm{Tr}^m(\alpha)$ for simplicity. For any set E, $\#E$ denotes the cardinality of E.

Let n and k be positive integers. A linear code $\mathcal{C}$ of length n and dimension k over $\mathbb{F}_p$ is a k-dimensional linear subspace of $\mathbb{F}_p^n$, denoted by $[n,k]_p$. Moreover, $\mathcal{C}$ is denoted by $[n,k,d]_p$ if its minimum Hamming distance d is known. A linear code $\mathcal{C}$ is *minimal* if every nonzero codeword $\mathbf{c}$ in $\mathcal{C}$ covers only the codewords $j\mathbf{c}$ for all $j \in \mathbb{F}_p$.

Lemma 22 (Ashikhmin-Barg Condition). [1] *Let $\mathcal{C}$ be a linear code over $\mathbb{F}_p$ and let $w_{\min}$ and $w_{\max}$ represent, respectively, the minimum and maximum Hamming weights of $\mathcal{C}$. Then, $\mathcal{C}$ is minimal if $\frac{p-1}{p} < \frac{w_{\min}}{w_{\max}}$.*

This work constructs new classes of three-weight and four-weight minimal linear codes. This work is motivated by the recent works [2,3]. In 2023, Zhu et. al. [2] have defined the following linear code

$$\mathcal{C}_D = \left\{ \mathbf{c}_{(a,b)} = (\mathrm{Tr}^m(ayx^t + bx))_{(x,y)\in D} \ : \ (a,b) \in \mathbb{F}_{p^m} \times \mathbb{F}_{p^m} \right\} \tag{7}$$

based on the defining set D for a positive integer t. The length of the code (7) is $n = \#D$ and its dimension is $k = 2m$. In this work, as a defining set D, we select the following set

$$D_{01} = \left\{ (x,y) \in \mathbb{F}_{p^m}^\star \times \mathbb{F}_{p^m} \ : \ \mathrm{Tr}^m(yx^{t+1}) \in \{0,1\} \right\}$$

for the code (7) and obtain a new class of three-weight linear code $\mathcal{C}_{D_{01}}$. Its minimality follows from Lemma 22.

The parameters of the code $\mathcal{C}_{D_{01}}$ are listed in the following theorem.

Theorem 23. *Let $m \geq 2$ be an integer. The code $\mathcal{C}_{D_{01}}$ is a three-weight minimal linear code over $\mathbb{F}_p$ with parameters $[2p^{2m-1} - 2p^{m-1}, 2m, p^{m-1}(2p^m - 2p^{m-1} - p + 1)]$. The Hamming weights are listed in Table 1.*

Hamming weight ω	Frequency A_ω
0	1
$(p-1)2p^{2m-2}$	$(\frac{1}{2}(p-1)p^{m-1}+1)(p^m-1)$
$p^{m-1}(2p^m - 2p^{m-1} - p + 1)$	$(p^m-1)p^{m-1}$
$2p^{m-1}(p^m - p^{m-1} - 1)$	$\frac{1}{2}(p-1)p^{m-1}(p^m-1)$

Table 1: Hamming weights and their frequency in $\mathcal{C}_{D_{01}}$

In 2022, Cheng et. al. [3] have defined the following linear code

$$\mathcal{C}_D = \{\mathbf{c}_{(a,b,c)} = (\mathrm{Tr}^m(ax+by+cz))_{(x,y,z)\in D} \ : \ (a,b,c) \in \mathbb{F}_{p^m}^3\}. \tag{8}$$

Motivated by the construction methods of (7) and (8), for an arbitrary positive integer t, we define a new linear code

$$\mathcal{C}_{D_0} = \left\{\mathbf{c}_{(a,b,c)} = (\mathrm{Tr}^m(ayx^t+bx+cz))_{(x,y,z)\in D_0} \ : \ (a,b,c) \in \mathbb{F}_{p^m}^3\right\}$$

based on the set $D_0 = \left\{(x,y,z) \in \mathbb{F}_{p^m}^\star \times \mathbb{F}_{p^m} \times \mathbb{F}_{p^m} \ : \ \mathrm{Tr}^m(yx^{t+1}) + \mathrm{Tr}^m(z) = 0\right\}$. The length $n = \#D_0$ and dimension $k = 3m$. Thus, we obtain a new class of four-weight linear code $\mathcal{C}_{D_0}$ whose minimality follows from Lemma 22.
The parameters of the code $\mathcal{C}_{D_0}$ are listed in the following theorem.

Theorem 24. *Let $m \geq 2$ be an integer. Then, $\mathcal{C}_{D_0}$ is a four-weight minimal linear code over $\mathbb{F}_p$ with parameters $[p^{2m-1}(p^m-1), 3m, (p-1)(p^{3m-2}-2p^{2m-2})]$. The Hamming weights are listed in Table 2.*

Hamming weight ω	Frequency A_ω
0	1
$(p-1)(p^m-1)p^{2m-2}$	$2p^{2m}-2p^m$
$(p-1)p^{3m-2}$	p^m-1
$(p-1)(p^{3m-2}-2p^{2m-2})$	$(p^m-1)^2p^{m-1}$
$(p-1)p^{3m-2}-(p-2)p^{2m-2}$	$(p^m-1)^2p^{m-1}(p-1)$

Table 2: Hamming weights and their frequency in $\mathcal{C}_{D_0}$

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General area of research: Coding Theory
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Contribution Talks

Co-commuting generalized derivations acting on Lie ideals in prime rings

Basudeb Dhara

Department of Mathematics, Belda College, Belda, Paschim Medinipur, India; basu_dhara@yahoo.com

Let R be a prime ring with its Utumi ring of quotients U and extended centroid C. Suppose that F, G and H are three generalized derivations of R and L is a noncentral Lie ideal of R such that

$$\Big(F(u)u - uG(u)\Big)H(u) = 0$$

for all $u \in L$. If char $(R) \neq 2, 3$, then one of the following holds:

1. $H = 0$;
2. there exists $a \in U$ such that $F(x) = xa$ and $G(x) = ax$ for all $x \in R$;
3. there exist $p, q, c \in U$ and $\lambda \in C$ such that $F(x) = xp + \lambda x$, $G(x) = px + xq$, $H(x) = cx$ for all $x \in R$, with $(\lambda - q)c = 0$;
4. R satisfies s_4 and one of the following holds:
   (a) there exist $a, p \in U$ and $\lambda \in C$ such that $F(x) = ax+xp+\lambda x$ and $G(x) = px+xa+\lambda x$ for all $x \in R$;
   (b) there exist $a, c, p, q \in U$ and $\lambda \in C$ such that $F(x) = ax + xp + \lambda x$, $G(x) = px + xq$, $H(x) = cx$ for all $x \in R$ with $(a - q + \lambda)c = 0$.

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Keywords: Prime ring, Derivation, Generalized derivation, Extended centroid, Utumi quotient ring, Lie ideal
General area of research: Generalized derivations
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Computing the Sombor Index of Prime Ideal Sum Graph of $\mathbb{Z}_n$

Elif Eryaşar[1], Esra Öztürk Sözen[2]

[1]Department of Mathematics, Sinop University, Sinop, Türkiye; eeryasar@sinop.edu.tr
[2]Department of Mathematics, Sinop University, Sinop, Türkiye; esozen@sinop.edu.tr

Algebraic graph theory is a significant field of mathematics that investigates the relationships between different algebraic structures and the numerous features that graphs display. Topological indices are used to represent graph structures numerically. In this study, Sombor index of the prime ideal sum graph of $\mathbb{Z}_n$ are calculated for $n = p^\alpha, p^2q, p^2q^2, p^3q, pqr$, where p, q and r are distinct primes. Finally, an algorithm is presented for calculating Sombor index of prime ideal sum graph structures for any positive n integer value in $\mathbb{Z}_n$.

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Keywords: Algebraic graph theory, prime ideal sum graph, Sombor index
General area of research: Algebraic Combinatorics and Applications
IIIMT25-ID 1019

Additive maps having nilpotent values on prime and semiprime rings

Giovanni Scudo[1]

[1] Department of Engineering, University of Messina, Messina, Italy; gscudo@unime.it

Starting from well-known results in literature (see for istance the results contained in [1], [2] and [3]), it is possible to study the structure of associative rings that satisfy suitable nilpotent conditions, which involve appropriate additive maps.
The most recent results have confirmed that this line of research can be conducted using the tools of functional identities.

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Keywords: Derivation, prime ring, Lie ideal
General area of research: Non-commutative Algebra
IIIMT25-ID 1020

Periodic Values of Generalized Skew Derivations in Prime Rings

Milena Andaloro[1]

[1] MIFT, University of Messina, 98166, Messina, Italy; milena.andaloro@studenti.unime.it

Let R be an associative ring. Several papers in literature are devoted to the study of an additive map $F : R \longrightarrow R$ satisfying the relation $F(x)^n = F(x)$, for all x in a suitable subset S of R (we refer to [1-7]). After recalling some classical results relating to this research area, we will present some new and recent ones. In particular, we will provide a detailed description of the structure of a prime ring R, in the case F is a generalized skew derivation of R.

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Keywords: Prime ring, generalized skew derivation
General area of research: Non-commutative algebra
IIIMT25-ID 1021

Idealization of Γ-Modules and Its Properties

Shadi Shaqaqha

Department of Mathematics, Yarmouk University, Shafiq Irshidat Street, Irbid 21163, Irbid, Jordan; shadi.s@yu.edu.jo

The concept of idealization, introduced by Nagata, extends a ring by incorporating a module as an ideal, enriching the underlying algebraic structure. In this work, we extend this construction to Γ-rings by introducing the idealization process for Γ-modules. We establish fundamental properties of the idealization $R \rtimes M$, proving that it inherits the structure of a Γ-ring while preserving the module's identity as a Γ-ideal. Furthermore, we investigate conditions under which the idealization retains essential algebraic properties such as commutativity and the existence of identity elements. The interaction between Γ-ideals and Γ-submodules in this framework is analyzed, providing structural insights into their behavior under idealization. Our results offer a new perspective on extending classical idealization techniques to the broader setting of Γ-rings, with potential implications for graded and nonassociative algebraic structures.

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Keywords: Γ-modules, Γ-rings, idealization, Γ-ideals
General area of research: Algebra (Groups, Rings, Lie algebras and Related Topics
IIIMT25-ID 1025

On a result on b-generalized derivations of prime rings

Nihan Baydar Yarbil[1]

[1]Department of Mathematics, Ege University , İzmir, Türkiye; nihan.baydar.yarbil@ege.edu.tr

The characterization of additive mappings defined on rings has long been a subject of interest for distinguished researchers. As well known by many working in this field, when studying rings satisfying identities involving additive mappings, the primary objective is to determine the structure of the map. In cases where this is not possible, the focus shifts to deriving certain structural conclusions about the ring itself. Numerous studies on derivations have been extended to generalized derivations, and research on generalized derivations has further expanded to generalized skew derivations and b-generalized derivations. In these studies on rings with identities, the techniques from the theory of rings with (generalized) polynomial identities serve as the primary methods leading researchers to their conclusions.([1],[2],[3],[4])

Definition: Let R be a ring and $Q_{mr}(R)$ be its right maksimal ring of quotients and $d : R \to Q_{mr}(R)$ be an additive map. An additive map $F : R \to Q_{mr}(R)$ is called a b-generalized derivation with the associated map d, if $F(xy) = F(x)y + bxd(y)$ for all $x, y \in R$. ([5])

In light of all these motivations, the study conducted by Pandey in [6] on generalized derivations has been extended to b-generalized derivations.

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https://doi.org/10.1142/S179355712350002X.

Keywords: Prime rings, generalized polynomial identity, differential identitiy, Martindale ring of Quotients

General area of research: Ring Theory

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Paper Presentations

Sombor Energy of the Zero Divisor Graph for the Ring of Integer Modulo pq

Nur'Ain Adriana Mohd Rizal[1], Nur Idayu Alimon[2], Mathuri Selvarajoo[3], Nor Haniza Sarmin[4]

[1]Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Johor Branch, Segamat Campus, 85000 Segamat, Johor, Malaysia; nurainadrianarizal@gmail.com
[2]Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Johor Branch, Pasir Gudang Campus, 81750 Masai, Johor, Malaysia; idayualimon@uitm.edu.my
[3]Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Shah Alam, 40450 Shah Alam, Malaysia; mathuri644@uitm.edu.my
[4]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, Johor, Malaysia; nhs@utm.my

Abstract

The Sombor energy of a graph was introduced in 2021. It is defined as the total sum of the absolute values of the eigenvalues of the Sombor matrix associated to a graph. The diagonal entries of the Sombor matrix are zero, and for any two adjacent vertices in the graph, the corresponding matrix entry depends on their degrees. If two vertices are not adjacent, their matrix entry is zero. Additionally, the zero divisor graph of a commutative ring is a graph that consists of nonzero zero divisors of the ring as the set of vertices, where two vertices are adjacent if and only if their product is zero and they commute. In this paper, the Sombor energy of the zero divisor graph for the ring of the integer modulo pq is computed and a general formula of the Sombor energy is established.

Keywords: Sombor matrix, Sombor polynomial, Sombor energy, zero divisor graph.

Introduction

The concept of energy was initially introduced by Gutman [1], where it is calculated as the total of all positive eigenvalues derived from the adjacency matrix, representing the graph's spectrum [1]. This notion was inspired by the Hückel Molecular Orbital (HMO) Theory of the 1930s, which estimates the energies of π-electron orbitals in conjugated hydrocarbon molecules. Recently, Gutman [2] proposed a new topological index in chemical graph theory, known as the Sombor index, based on vertex degrees. Building on this foundation, Gowtham and Narasimha [3] further defined Sombor energy, a new type of graph energy, and introduced the Sombor matrix as a representation of the graphs.

Subsequently, many researchers have explored and expanded the concept of Sombor energy. For instance, Singh and Patekar [4] derived a general formula for the Sombor index of m-splitting and m-shadow graphs and determined the relationship between energy and Sombor energy for m-splitting and m-shadow graphs of k-regular graphs.

Meanwhile, the idea of zero divisor graph was introduced by Beck [5] where the study focuses on colorings of commutative rings. From Beck's work, Anderson and Livingston [6] then found a slightly altered definition of the zero divisor graph of commutative rings. Later, Magi et al [7] in 2020 investigated the characteristic polynomial and generalizes methods for determining the spectrum of zero divisor graphs and Semil et al [8] generalized the formula of first Zagreb index of the zero divisor graph for the commutative ring, $\mathbb{Z}_{p^k}$.

This paper focuses on Sombor energy of zero divisor graph for the commutative ring of integer modulo pq. The first section of this paper is an introduction, followed by Preliminaries where some basic concepts and definitions on ring theory, graph theory and energy are stated. In the last section, the main results of the Sombor energy of zero divisor graph for the ring integer modulo pq are presented.

Preliminaries

In this section, the definitions and the basic ideas of energy, graph theory, and ring theory are presented. The following definition states the concept of a zero divisor graph within the context of ring theory.

Definition 1 [6] Zero Divisor Graph of Commutative Rings
Let $\Gamma(Z(R))$ represent the zero divisor graph of a commutative ring R. The vertices of the graph are the nonzero zero divisors of R, and two vertices a and b are connected if and only if $ab = ba = 0$.

Definition 2 [2] Sombor Index
Let Γ be the simple undirected graph with a vertex set $V(\Gamma)$ and edge set $E(\Gamma)$. The Sombor index of Γ denoted by $SI(\Gamma)$, is defined as

$$SI(\Gamma) = \sum_{e_{ij} \in E(\Gamma)} \sqrt{\deg(v_i)^2 + \deg(v_j)^2}$$

where $deg(v_i)$ and $deg(v_j)$ denotes the degree of the vertices v_i and v_j respectively and e_{ij} is the edge connecting v_i and v_j in Γ.

Based on the definition of the Sombor index, a corresponding matrix representation, known as the Sombor matrix, was developed. For each pair of adjacent vertices, the corresponding matrix entry is defined using the Sombor index formula $\sqrt{\deg(v_i)^2 + \deg(v_j)^2}$, while entries for non-adjacent vertices are zero. The Sombor matrix is formally defined as follows.

Definition 3 [3] Sombor Matrix
The Sombor matrix of a graph Γ, $SO(\Gamma)$, with vertex set, $V(\Gamma)$, and edge set, $E(\Gamma)$, is defined such that $SO_{ii} = 0$, $SOij = \sqrt{deg(v_i)^2 + deg(v_j)^2}$ if vertices v_i and v_j are adjacent, and $SO_{ij} = 0$ otherwise, where $deg(v_i)$ and $deg(v_j)$ are the degrees of vertices v_i and v_j respectively.

Definition 4 [3] Sombor Energy of a Graph
For a given Γ, the Sombor matrix $SO(\Gamma)$ is a real symmetric matrix, which means all the eigenvalues of this matrix are real numbers. These eigenvalues can be ordered from largest to smallest as $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ where n denotes the total the total number of eigenvalues. The Sombor energy, $SE(\Gamma)$, of the graph is then defined as the total sum of the absolute values of these eigenvalues:

$$SE(\Gamma) = \sum_{i=1}^{n} |\lambda_i|.$$

Main Results

In this section, the Sombor energy of zero divisor graph for the ring of integer modulo pq where p, q are primes and $p \neq q$ are shown in the main theorems.

Theorem 1 Let Γ be the zero divisor graph of the commutative ring $\mathbb{Z}_{pq}$. Then, the Sombor eigenvalues of $\Gamma(\mathbb{Z}_{pq})$ are 0 with multiplicity $q-1$, $\sqrt{2(q-1)[(p-1)^2 + (q-1)^2]}$, and $-\sqrt{2(q-1)[(p-1)^2 + (q-1)^2]}$ where p, q are primes and $p \neq q$.

Theorem 2 Let Γ be the zero divisor graph for commutative ring $\mathbb{Z}_{pq}$. The Sombor energy of $\Gamma(\mathbb{Z}_{pq})$ is $SE(\Gamma) = 2\sqrt{2(q-1)[(p-1)^2 + (q-1)^2]}$ where p, q are primes and $p \neq q$.

Conclusions

In this paper, the generalization of the Sombor energy of zero divisor graph for the ring of integer modulo pq is determined. The general formulas found demonstrates its adaptability in exploring algebraic structures, with potential applications extending to molecular chemistry, as Sombor energy has been proven useful in modeling thermo dynamic properties of molecular graphs [9]. Moreover, other types of energies, such as Seidel energy, distance energy, or signless Laplacian energy of the zero divisor graph for some commutative rings, can be determined for future studies.

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Keywords: Sombor energy, graphs, rings **General area of research:** Algebraic Graph Theory
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Topological Indices: The Zagreb and Randić Indices of The Clean Graph Over $\mathbb{Z}_n$

Felicia Servina Djuang[1], Indah Emilia Wijayanti[2], Yeni Susanti[3]

[1]Department of Mathematics, Universitas Gadjah Mada, Yogyakarta, Indonesia; feliciadjuang25@mail.ugm.ac.id
[2]Department of Mathematics, Universitas Gadjah Mada, Yogyakarta, Indonesia; ind_wijayanti@ugm.ac.id
[3]Department of Mathematics, Universitas Gadjah Mada, Yogyakarta, Indonesia; yeni_math@ugm.ac.id

Let R be a finite ring with identity. The clean graph $Cl(R)$ of a ring R is a graph whose vertices are of the form (e, u), where e is an idempotent element and u is a unit of R. Two distinct vertices (e, u) and (f, v) are adjacent if and only if $ef = fe = 0$ or $uv = vu = 1$. The graph $Cl_2(R)$ is the subgraph of $Cl(R)$ induced by the set $\{(e, u) : e \text{ is a nonzero idempotent element of } R\}$. In this study, we examine the Zagreb and Randić topological indices of clean graphs $Cl_2(\mathbb{Z}_n)$. We also present the condition such that two clean graph over direct product two ring are isomorphic.

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Keywords: clean graph, idempotent, unit, isomorphism graph, topological indices.

General area of research: Algebra
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Some Properties of $\mathscr{D}$-sets in Infinite Groups

Puspa Nur Afifah[1], Irawati[2]

[1]Department of Mathematics, Universitas Islam Negeri Sultan Maulana Hasanuddin Banten, Indonesia; puspa.nurafifah@uinbanten.ac.id
[2] Ph.D. Graduate in Mathematics, Institut Teknologi Bandung, Indonesia; yoshua.yonatan.h@gmail.com

Concept of $\mathscr{D}$-Sets infinite groups has been widely studied, particularly in determining their minimal size based on the structure of involutions and inverse pairs. However, extending this concept to infinite groups introduces significant challenges. This research investigates the existence of infinite minimal $\mathscr{D}$-Sets in infinite groups. We prove that such a set exists if and only if the group has infinite number of involutions, is locally finite, and contains a finite subset with a closed invers relation. Furthermore, we introduce the notion of locally finite $\mathscr{D}$-Sets to generalize the theory of $\mathscr{D}$-Sets in infinite groups. To illustrate our findings, we provide examples including the integer orthogonal group. These results extend classical properties of $\mathscr{D}$-sets and offer new insights into their behavior within infinite groups, especially under finiteness conditions.

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Keywords: $\mathscr{D}$-Set, infinite group, minimal $\mathscr{D}$-Set, involution, locally finite group.
General area of research: Group Theory
IIIMT25-ID 1018

Symmetric Reverse n-Derivations and Functional Identities in Algebraic Structures

Shakir Ali[1] and Nadia Farj[2]

[1]Aligarh Muslim University, Aligarh, India; drshakir1971@gmail.com
[2]University of Prince Mugrin, Saudi Arabia, Al-Madinah Al-Munawwarah; n.alrehele@gmail.com

The main purpose of this paper is to investigate the structure of symmetric reverse n-derivations that satisfying some functional identities (FIs) in the setting of semiprime rings and algebras (cf. [2-5] for details). The study examines whether these derivations impose new constraints, lead to triviality, or contribute to additional algebraic structure. The findings aim to enhance the understanding of functional identities involving semiprime rings.

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Keywords: Functional Identities (FIs); semiprime ring; derivation; symmetric n-derivation; n-multiplier
General area of research: Algebra
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Derivations with symmetric elements in prime rings

Hiba Fihi[1], Abdellah Mamouni[2], Khalid Ouarghi[3]

[1]Department of Mathematics, Faculty of Sciences, University of Moulay Ismaïl, Meknes, Morocco; hiba.fihi@usmba.ac.ma
[2]Department of Mathematics, Faculty of Sciences, University of Moulay Ismaïl, Meknes, Morocco; a.mamouni.fste@gmail.com
[3]Department of Mathematics, Ecole Normale Superieure, University of Moulay Ismaïl, Meknes; Morocco; ouarghi.khalid@hotmail.fr

ABSTRACT
In this paper, we investigate the structure of Hermitian elements in prime rings with involution *, characterized by specific commutativity conditions involving derivations. We establish that these elements are either central or have squares that belong to the center. Additionally, we extend these results to skew-Hermitian elements, demonstrating similar centrality properties.

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General area of research: Algebra, Group Theory, Derivatives
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A Decomposition of Symmetric Numerical Semigroups

Meral Süer[1], Mehmet Yeşil[2]

[1]Department of Mathematics, Batman University, Batman, Türkiye; meral.suer@batman.edu.tr
[2]Department of Mathematics, Batman University, Batman, Türkiye; mehmet-yesil@outlook.com

Let $\mathbb{N}$ be the set of non-negative integers. A numerical semigroup S is a subset of $\mathbb{N}$ that is closed under addition, contains zero, and has finite complement in $\mathbb{N}$. If $n_1, \ldots, n_e$ are positive integers with $\gcd(n_1, \ldots, n_e) = 1$, then the set $\langle n_1, \ldots, n_e \rangle = \{a_1 n_1 + \cdots + a_e n_e \mid a_1, \ldots, a_e \in \mathbb{N}\}$ is a numerical semigroup, and every numerical semigroup is of this form.
Numerical semigroups play a significant role in commutative algebra and algebraic geometry. Let $S = \langle n_1, \ldots, n_e \rangle$ be a numerical semigroup and $\mathbb{K}$ be a field. Let $R = \mathbb{K}[x^{n_1}, \ldots, x^{n_e}]$ be $\mathbb{K}$-algebra of polynomials $x^{n_1}, \ldots, x^{n_e}$. The ring R is the coordinate ring of the curve parametrized by $x^{n_1}, \ldots, x^{n_e}$ and and information from R can be derived from the properties of S. In this regard, one of the most important results is by Kunz stating that R is a Gorenstein ring if and only if S is a symmetric numerical semigroup.
A Young diagram is a series of left aligned rows of unit boxes such that the number of boxes in each row is not less than the number of boxes in the row immediately below it. Numerical semigroups can be visualised with Young diagrams, i.e one can always draw a unique Young diagram for a given numerical semigroup.
Symmetric numerical semigroups is one of the main classes amongst the others. They are known as irreducible in the usual sense. That is, a symmetric numerical semigroup can not be written as an intersection of some other numerical semigroups containing it. In this talk, we will introduce a method for decomposing a symmetric numerical semigroup into a numerical semigroup containing it and its dual using the corresponding Young diagrams.

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Keywords: Numerical sets, Young diagrams, Symmetric numerical semigroups
General area of research: Commutative Algebra, Numerical Semigroups
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A Frobenius Ring-based Signature Scheme through Constacyclic Codes

Asmaa Cherkaoui[1], Seddik Abdelalim[1], Abdelkarim Lkoaiza[1], Ilias Elmouki[1]

[1]Laboratory of Fundamental Mathematics and Applications (LMFA), Faculty of Sciences Ain Chock (FSAC), Hassan II University of Casablanca, Casablanca, Morocco.

In this work, we study an example of post-quantum code-based signature schemes, namely the Linear Equivalence Signature Scheme (LESS). Then, we investigate how we could adapt this scheme to operate over the Frobenius ring $R = \mathbb{F}_q + u\mathbb{F}_q + v\mathbb{F}_q + uv\mathbb{F}_q$, where $u^2 = u$, $v^2 = v$, and $uv = vu$, while relying on constacyclic codes for which we explore their interaction with the present modified version of the Linear Code Equivalence (LCE) problem. Finally, we show how our results could be beneficial in understanding some key elements for the development of a robust ring-based post-quantum signature scheme.

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Keywords: Constacyclic codes, Frobenius ring, Code equivalence problem
General area of research: Ring theory, Coding theory, Post-Quantum Cryptography
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A Note on Hartshorne Theorems and Properties of Isogenies with Applications

Ilias Elmouki[1], Seddik Abdelalim[1]

[1]Laboratory of Fundamental and Applied Mathematics, Faculty of Sciences Ain Chock, University Hassan II of Casablanca, Casablanca, Morocco; i.elmouki@gmail.com, seddikabd@hotmail.com

We aim from this work to discuss the properties of isogenies after first revisiting some Hartshorne theorems. In fact, some aspects of ring homomorphisms in the context of elliptic curves have not been sufficiently studied because with the testimony of specialists in algebraic geometry, they are not that easy to prove. Finally, we provide practical examples of our approach while presenting promising applications of isogenies.

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Keywords: Nonsingularity, Projective curve, Isogeny, Elliptic curve
General area of research: Algebraic Geometry
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A Gröbner basis attack on generalized Grendel-based hashing with application to blockchainsignatures

Abdelkarim Lkoaiza[1], Seddik Abdelalim[1], Asmaa Cherkaoui[1], Ilias Elmouki [1]

[1]Laboratory of Fundamental Mathematics and Applications (LMFA),Faculty of Sciences Ain Chock (FSAC), Hassan II University of Casablanca, Casablanca, Morocco; karimlkoaiza6@gmail.com, seddikabd@hotmail.com, esma1maysan@gmail.com, i.elmouki@gmail.com

In this paper, we study an attack against a new hashing model through the introduction of the Gröbner basis. The hashing process here relies on our generalized Grendel-based hashing which incorporates modified Legendre symbols L_{pq}. In fact, the algebraic properties of our method helps to improve preimage and collision resistance. As for the practical part of our work, we provide an example of our Gröbner-based attack and we analyze its complexity. Finally, we show the possible contribution of our approach to blockchainsignatures.

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Keywords: Grendel hashing, Blockchain signature, Keccak family, Sponge function
General area of research: Mathematical Arithmetic, Cryptography
IIIMT25-ID 1036

On the Strong Persistence Property of Some Classes of Monomial Ideals

Hafsa Bibi[1], Hanni Garminia Y.[2], Irawati[3]

[1]Mathematics Department, Institute of Technology, Bandung, Indonesia; hafsaliaqat600@gmail.com
[2]Mathematics Department, Institute of Technology, Bandung, Indonesia; garminia@itb.ac.id
[3]Mathematics Department, Institute of Technology, Bandung, Indonesia; irawati@itb.ac.id

Monomial ideal plays an important role in combinatorial commutative algebra. In commutative Noetherian ring R, the associated primes are connected to the primary decomposition of ideals. All monomial ideals need not to hold the persistence property and strong persistence property. There are some known classes of monomial ideals that satisfy the strong persistence property. These classes are edge ideals of a simple graph, edge ideals of a graph with loops, vertex cover ideal of perfect graphs, vertex cover ideals of cycle graphs of odd orders, vertex cover ideal of wheel graphs of even orders, all square-free monomial ideals in R with $n \leq 4$, irreducible ideal and every normally torsion-free square-free monomial ideals. In a polynomial ring R, persistence property, strong persistence property, and stable set of associated primes of different classes of monomial ideals will be discussed.

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Keywords: Associated prime ideals, Persistence property, Strong persistence property
General area of research: Commutative Algebra
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Developing a Lattice Reduction through Householder and Givens Orthogonalization Processes

Mohammed-Yasser Ez-zaher[1], Seddik Abdelalim[1], Ilias Elmouki[1]

[1]Laboratory of Fundamental and Applied Mathematics, Faculty of Sciences Ain Chock, University Hassan II of Casablanca, Casablanca, Morocco; ez.yasser@gmail.com, seddikabd@hotmail.com, i.elmouki@gmail.com,

The present study discusses the essentials of the reduction of the lattice basis and the possible development of methods that could serve to devise analogous versions of the Lenstra-Lenstra-Lovász (LLL) and the Abdelalim-Elmouki (AE) algorithms. Then, we state and prove results on properties of the lattice reduced basis while relying on new algorithmic approaches that introduce Householder and Givens orthogonalization procedures. Finally, we present our numerical results and provide examples of applications.

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Keywords: Lattice basis reduction, Gram-schmidt process, Householder process, Givens Process, LLL algorithm, AE algorithm
General area of research: Bilinear Algebra, Lattice reduction
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Köethe Conjecture Revisited: an Introduction to Quasi Reduced Rings

Puguh Wahyu Prasetyo[1], Indah Emilia Wijayanti[2], Joe Repka[3]

[1]Mathematics Education Department, Faculty of Teacher Training, Universitas Ahmad Dahlan, Main Campus, South Ringroad, Tamanan, Bantul, Yogyakarta Special District, Indonesia 55191, 7th Floor, Main Building, Universitas Ahmad Dahlan Main Campus; puguh.prasetyo@pmat.uad.ac.id
[2]Mathematics Department, Faculty of Mathematics and Sciences, Universitas Gadjah Mada Bulaksumur, Sleman, Yogyakarta Special District, Indonesia 55281; ind_wijayanti@ugm.ac.id
[3]Department of Mathematics, University of Toronto, Bahen Centre, Room 6290, 40 St. George St., Toronto, Ontario, M5S 2E4, Canada; repka@math.toronto.edu

The Köethe conjecture, originally proposed by Gottfried Köethe in 1930, asserts that in any ring, the sum of a nilpotent subring and a nil subring is itself nil. This conjecture has been verified for several important classes of rings, such as all right Noetherian rings and quasi 2-primal rings. In this paper, we focus on 2-primal rings, weakly 2-primal rings, and quasi 2-primal rings as central objects of study. We introduce the notion of a quasi reduced ring, which generalizes the concept of a reduced ring, and demonstrate that quasi reduced rings also satisfy the Köethe conjecture.

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Keywords: Köethe conjecture, reduced ring, quasi reduced ring
General area of research: Radical Theory of Rings
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An Example of a Ring Extending from Abstract Algebra to Gene Algebra

Nazlıcan Kümbet[1], Esra Öztürk Sözen[2]

[1]Department of Mathematics, Sinop University, Sinop, Türkiye; nazlicankumbet226@gmail.com
[2]Department of Mathematics, Sinop University, Sinop, Türkiye; esozen@sinop.edu.tr

In the minds of undergraduate students of the Department of Mathematics, the most important question mark regarding mathematics is where they will encounter the theoretical knowledge given to them in their real lives and how they will use it. On the other hand, although they know that the teachings of each sub-branch of mathematics form the basis for innovations in the fields of science, engineering, health, etc., due to the limitations of the department curriculum in terms of application, they cannot go beyond remaining in their minds as a sentence form only. For example, the differential equation of blood circulation in the veins was determined with the mathematical modeling constructed by Euler. Thanks to other models established in light of this, the way for developments in the diagnosis of heart, kidney, pancreas and ear diseases has been paved.
In this presentation (supported by TUBITAK 2209-A /2024-I), the axioms that enable the construction of the Primal Codon Group and the ring structure, which were created by algebraic modeling, will be examined. Thus, the contributions of a theoretically based algebraic structure to practice in a discipline such as molecular biology will be learned. These findings may open new corridors of study regarding the research on which other studies the modeling done specifically for Algebra has been used and can be used in other disciplines. Most importantly, the awareness of the students who encounter applied examples of mathematical modeling that mathematics is not confined to theoretical castles and that each piece of information has the potential to shed light on an innovative idea in the applied field will develop.

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Keywords: Group theory, Ring Theory, Primal Codon Group
General area of research: Abstract Algebra and Applications
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Exponential Sombor index and some of its notable features

Büşra AYDIN[1], Nihat AKGÜNEŞ[2], Sedat PAK[3]

[1]Department of Mathematics, Necmettin Erbakan University, Konya, Turkey; bsrcgn@gmail.com
[2]Department of Mathematics, Necmettin Erbakan University, Konya, Turkey; nakgunes@erbakan.edu.tr
[3]Department of Mathematics, Karamanoğlu Mehmetbey University, Karaman, Turkey; sedatpak@kmu.edu.tr

Recently, a novel class of topological molecular descriptors, referred to as the Sombor index and centered on degrees, has emerged. This index has rapidly gained recognition among its topological counterparts, especially with the pre-exiasting popularity of exponential topological índices. Within the study, we introduce concept of the exponential Sombor index and delve into the identification of extremal graphs. Additionally, we obtain the exponential sombor index values of some algebraic graphs depending on the parameters of their graphs.

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Keywords: exponential Sombor index, graph theory, Sombor index
General area of research: Algebra
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